The non-linear behavior of resistances in an inductance box

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1 Introduction

The goal of this project is to investigate the non-linear behavior of resistances in an inductance box, a common tool in electronic laboratories. The inductance box contains multiple inductive elements arranged in 7 columns and 4 rows, with switches to activate each element individually. When measuring the resistance, it's been observed that the total resistance of the activated inductances does not add up as expected, revealing a non-linear effect.

This project aims to analyze the resistance behavior and develop both statistical and physical models to explain these effects and predict the resistance for any combination of activated switches. Additionally, we will explore the relationship between the statistical and physical models. As the next step, we will test another inductance box and compare the results.

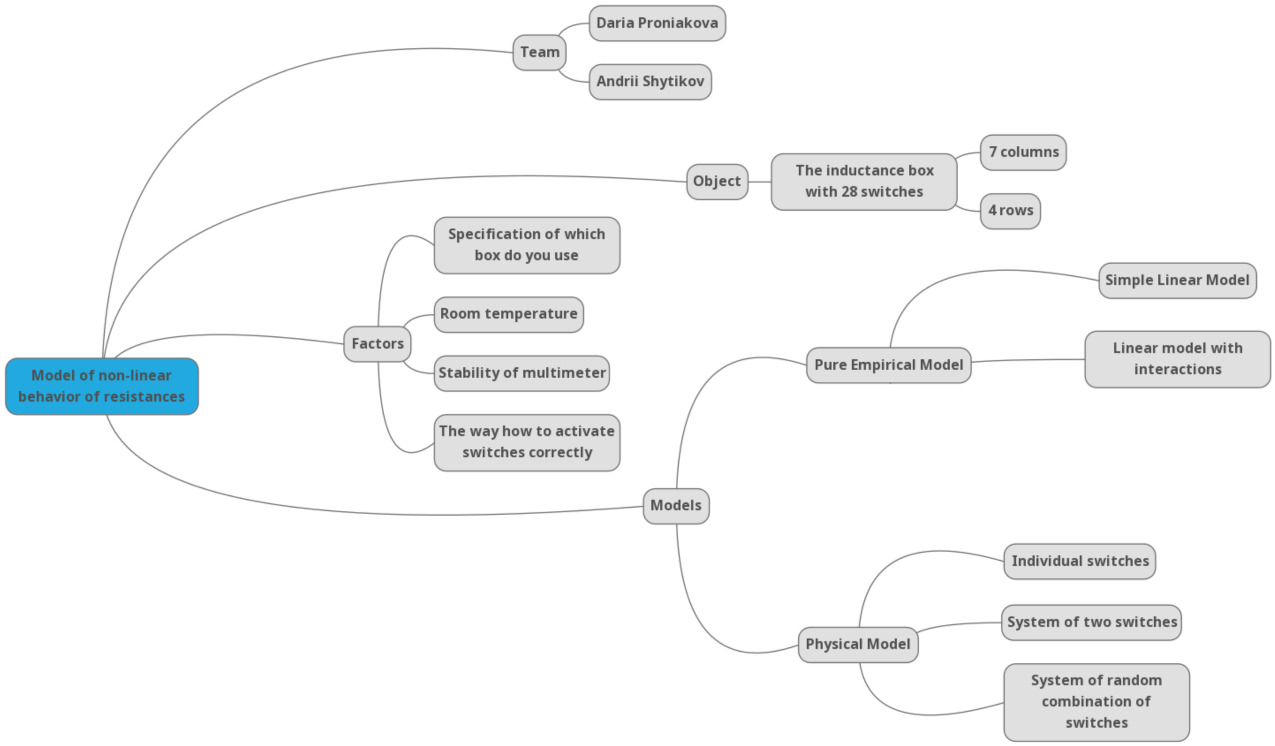

Figure 1: Mindmap to represent the main ideas of this project

Several factors can affect the experiment's outcomes:

- Room Temperature: Environmental conditions may influence resistance measurements.
- Multimeter Stability: The accuracy of the measuring device is key to reliable data.
- Switch Activation Method: How switches are turned on or off can introduce variability in the measurements.
- Specification of the Inductance Box: Differences in the characteristics of the inductance box can lead to variations in the experiment. It is important to account for the specific box being used.

To check these influences, we repeated the same switch combinations on different days. The results varied by less than the offset, 0.7 Ohms, which is much smaller than the measured resistances (tens of Ohms), so these factors can be ignored.

2 Individual switches

The first step in the scheme reconstruction process is the measurement of individual resistances (turning on switches one by one). The following values were obtained:

4 µH <1 Ω	40 µH <1 Ω	0.4 mH <1 Ω	4 mH 3.50 Ω	40 mH 4.10 Ω	0.4 H 33.20 Ω	4 H 67.10 Ω
3 µH <1 Ω	30 µH <1 Ω	0.3 mH <1 Ω	3 mH 2.80 Ω	30 mH 3.50 Ω	0.3 H 28.20 Ω	3 H 56.70 Ω
2 µH <1 Ω	20 µH <1 Ω	0.2 mH <1 Ω	2 mH 1.90 Ω	20 mH 3.20 Ω	0.2 H 16.80 Ω	2 H 45.40 Ω
1 µH <1 Ω	10 µH <1 Ω	0.1 mH <1 Ω	1 mH 1.50 Ω	10 mH 2.30 Ω	0.1 H 11.50 Ω	1 H 31.30 Ω

Table 1: The table presents the inductance values for each of the switches, as well as the measured resistances when each switch is individually activated.

The measurement of resistances for switches below 1 mH is not useful, as the measurement error (around 0.2–0.3 Ohms) and the offset, 0.7 Ohms (offset refers to the baseline resistance measurement when all switches are completely turned off). The use of small resistances in circuit reconstruction will result in significant errors. On the contrary, excluding them from consideration will not cause any problems for the device model creation.

3 System of two switches

Below are the tables showing the resistances in Ohms for the two-switch combinations. When comparing these values with individual switch measurements, it is important to remember that each measurement includes both the actual resistance and an offset. Therefore, 0.7 Ohms must be subtracted from each measurement to account for the offset.

	0	1 H	2 H	3 H	4 H
0					
1 H	31,30				
2 H	45,40	56,80			
3 H	56,70	87,60	101,50		
4 H	67,10	97,80	112,20	93,60	

	0	0.1 H	0.2 H	0.3 H	0.4 H
0					
0.1 H	11,50				
0.2 H	16,80	21,10			
0.3 H	28,20	39,00	44,30		
0.4 H	33,20	44,00	49,50	45,80	

	0	10 mH	20 mH	30 mH	40 mH
0					
10 mH	2,30				
20 mH	3,20	3,70			
30 mH	3,50	5,10	6,00		
40 mH	4,10	5,70	6,50	5,40	

	0	1 mH	2 mH	3 mH	4 mH
0					
1 mH	1,50				
2 mH	1,90	2,20			
3 mH	2,80	3,70	4,20		
4 mH	3,50	4,80	4,70	4,50	

We will number the switches within a single column as 1, 2, 3, 4, respectively, which could correspond to 1H, 2H, 3H, 4H, or 0.1H, 0.2H, 0.3H, 0.4H, or another column. The switch combinations within a column behave the same for all columns, which allows us to construct a general model applicable to each column.

The majority of switch combinations lead to simple summation of individual resistances: combinations such as 1+3, 1+4, 2+3, and 2+4 within the same column are summed additively.

Problems arise only for the combinations 1+2 and 3+4 in all columns.

3.1 Statistical Model

Let's make a model for switches 1H and 2H (The model for 3H and 4H will be the same, only with different coefficient values), this model should describe the interaction because at simultaneous switching on the total resistance differs from the sum of individual resistances.

$$y = a_0 + a_1 x_1 + a_2 x_2 + a_{12} x_1 x_2 \tag{1}$$

Where:

- y is the measured resistance,
- x_1 and x_2 are 0 for OFF and 1 for ON for the first and second switches respectively,
- a_0 is the intercept (baseline resistance),
- a_1 is the effect of switch 1H,
- a_2 is the effect of switch 2H,
- a_{12} is the interaction effect between the two switches.

The data from the experiments can be represented in matrix form. The design matrix X and the response vector Y are given by:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.7 \\ 31.3 \\ 45.4 \\ 56.8 \end{bmatrix}$$

The general formula for calculating the coefficients is:

$$\hat{\alpha} = (X^T X)^{-1} X^T Y = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_{12} \end{bmatrix} = \begin{bmatrix} 0.7 \\ 30.6 \\ 44.7 \\ -19.2 \end{bmatrix} \tag{2}$$

Thus, the final statistical model is:

$$y = 0.7 + 30.6 x_1 + 44.7 x_2 - 19.2 x_1 x_2 \tag{3}$$

This model demonstrates the negative interaction effect. This occurs because the total resistance, when both switches are ON, is lower than the sum of the individual resistances, indicating that the interaction term reduces the overall resistance compared to a simple additive (linear) model.

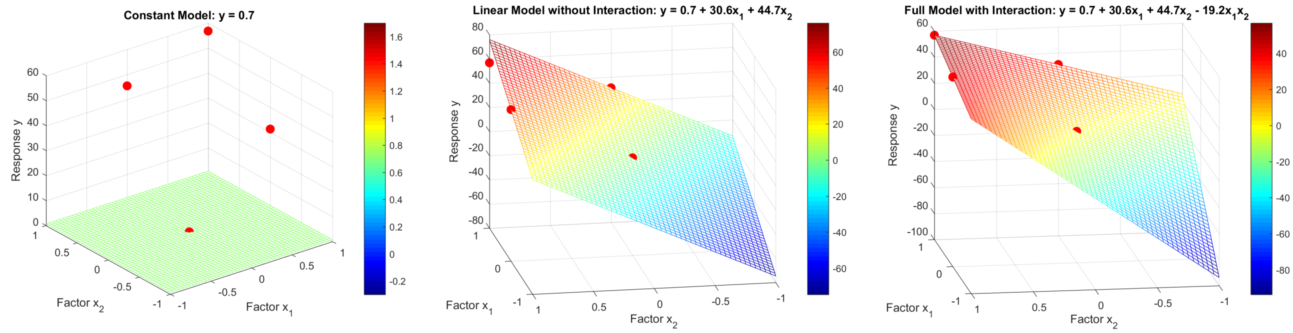

Figure 2: Comparison of different statistical models with marked experimental points.

Figure 2 shows that the constant model is too simplistic, assuming no effects from the activation of switches. The linear model shows independent, linear effects of each factor, without interactions or curvature. This model works for combinations with linear additive behaviour: 1+3, 1+4, 2+3, 2+4. The model with interaction gives the most accurate representation, providing a more complex but realistic response surface.

3.2 Physical Model

Let's begin reconstructing the electrical circuit with the simplest system of two switches, where the resistances are simply added together when both switches are activated. For example, consider switches 1H and 3H.:

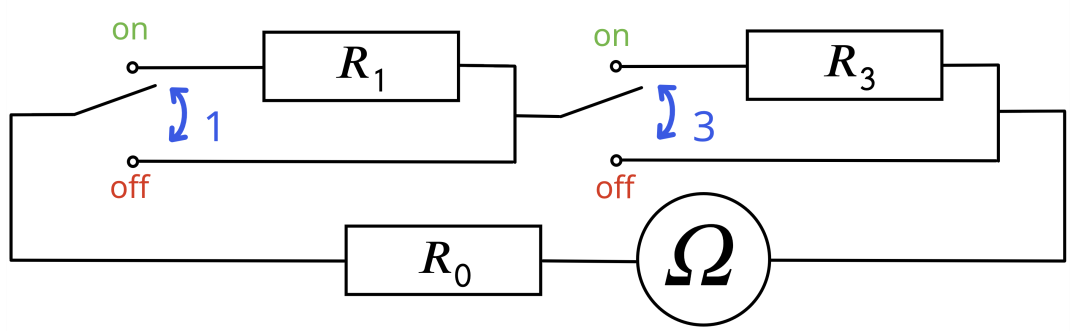

Figure 3: Scheme for the two-switch system, with simple resistance summing when both switches are turned on simultaneously. Ⓘ represents an ohmmeter.

- $(0,0) \rightarrow Y_0 = R_0 = 0.7\ \Omega$;
  When both switches are off, the result resistance is the offset R_0.
- $(1,0) \rightarrow Y_1 = R_0 + R_1 = 31.3\ \Omega$;
  When the first switch is on and the second is off, R_1 and R_0 are connected in series.
- $(0,1) \rightarrow Y_2 = R_0 + R_3 = 56.7\ \Omega$;
  When the third switch is on and the first is off, R_3 and R_0 are connected in series.
- $(1,1) \rightarrow Y_3 = R_0 + R_3 + R_1 = 87.6\ \Omega$;
  When both switches are on, R_3, R_1 and R_0 are connected in series. The model's prediction for $Y_3 = 31.3 + 56.7 - 0.7 = 87.3\ \Omega$, with an error of $0.3\ \Omega$ (we consider an error tolerable if it is smaller than the offset of $0.7\ \Omega$).

The main challenge is to design a circuit with two switches that explains the nonlinear behavior of the two switch systems 1 and 2 or 3 and 4.

The combination of switches 1+2 can be explained by this scheme:

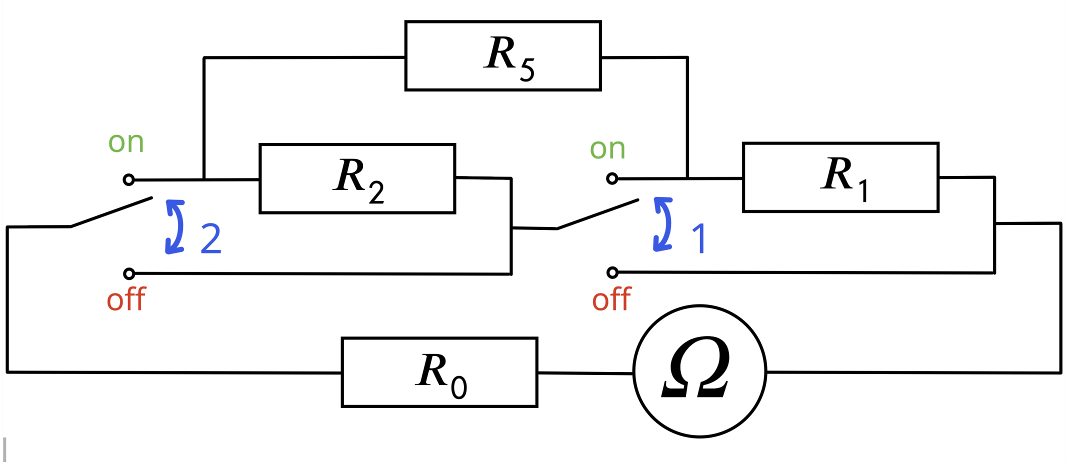

Figure 4: Scheme for the two-switch system with non-additive interaction, switches 1 and 2. An additional resistor was assigned number 5 to avoid confusion with the resistors associated with switches 3 and 4. Ⓘ represents an ohmmeter.

- $(0,0) \rightarrow Y_0 = R_0 = 0.7\ \Omega$;
  When both switches are off, the result resistance is the offset R_0.
- $(1,0) \rightarrow Y_1 = R_0 + R_1 = 31.3\ \Omega$;
  When the first switch is on and the second is off, R_1 and R_0 are connected in series.
- $(0,1) \rightarrow Y_2 = R_0 + \frac{R_2(R_1+R_5)}{R_1+R_2+R_5} = 45.4\ \Omega$;
  When the second switch is on and the first is off, R_1 and R_5 are connected in series, and R_2 is connected in parallel with $(R_1 + R_5)$.
- $(1,1) \rightarrow Y_3 = R_0 + R_1 + \frac{R_2 R_5}{R_2+R_5} = 56.8\ \Omega$;
  When both switches are on, R_2 and R_5 are connected in parallel, and R_1 is in series.

The solution for this system: $R_0 = 0.7\ \Omega$, $R_1 = 30.6\ \Omega$, $R_2 = 170.5\ \Omega$, $R_5 = 30\ \Omega$

The same logic applies to the combination of switches 3 and 4, it is only necessary to insert the values from the table.

3.3 Link Between the Two Models

Having both statistical and physical models, we can link them by deriving each coefficient in the statistical model using resistances from the physical model:

$$y = a_0 + a_1 x_1 + a_2 x_2 + a_{12} x_1 x_2$$

$$y = 0.7 + 30.6 x_1 + 44.7 x_2 - 19.2 x_1 x_2$$

$$a_0 = Y_0 = R_0 = 0.7$$

$$a_1 = Y_1 - Y_0 = R_1 = 30.6$$

$$a_2 = Y_2 - Y_0 = \frac{R_2 (R_1 + R_5)}{R_1 + R_2 + R_5} = 44.7$$

$$a_{12} = (Y_3 - Y_0) - ((Y_1 - Y_0) + (Y_2 - Y_0)) = \frac{R_2 R_5}{R_2 + R_5} - \frac{R_2 (R_1 + R_5)}{R_1 + R_2 + R_5} = -19.2$$

4 System of a random combination of switches

First, let's look at all the possible combinations for each of the columns

H	Ω
1,2,3	113,50
1,2,4	123,70
1,3,4	124,20
2,3,4	138,60
1,2,3,4	149,50

0.1 H	Ω
1,2,3	48,80
1,2,4	53,40
1,3,4	56,70
2,3,4	62,00
1,2,3,4	66,20

10 mH	Ω
1,2,3	6,50
1,2,4	7,10
1,3,4	7,00
2,3,4	7,70
1,2,3,4	8,30

mH	Ω
1,2,3	4,50
1,2,4	5,30
1,3,4	5,20
2,3,4	5,70
1,2,3,4	6,00

The already measured combinations (1+2) and (3+4) sum correctly (additively) with other switches. For instance, if switches 1, 2, and 3 are turned on, the result corresponds to the sum of the separate resistances of combinations (1+2) and 3. This simplifies the task of reconstructing the scheme significantly, as we only face issues with the 2-switch systems, while combinations of multiple switches behave well (i.e., simple additive summation).

Now, on the contrary, let's look at how the combinations between the columns behave within the selected row.

Combination	Ω
0.1H, 10mH	13,10
1H, 10mH	32,80
1H, 0.1H	42,10
1H, 0.1H, 10mH	43,70
1H, 0.1H, 10mH, 1mH	44,40

Combination	Ω
0.2H, 20mH	19,20
2H, 20mH	48,10
2H, 0.2H	61,70
2H, 0.2H, 20mH	64,20
2H, 0.2H, 20mH, 2mH	65,20

Combination	Ω
0.3H, 30mH	31,20
3H, 30mH	60,20
3H, 0.3H	85,00
3H, 0.3H, 30mH	88,00
3H, 0.3H, 30mH, 3mH	89,70

The column resistances sum up correctly (additively). Therefore, we can develop the schemes for each column separately and then connect them in series.

For final check we measure resistances of random combinations including all rows and columns.

Combination	Ω
0.4H, 40mH	36,70
4H, 40mH	71,00
4H, 0.4H	100,00
4H, 0.4H, 40mH	103,40
4H, 0.4H, 40mH, 4mH	106,00

Combination	Ω
0.2H, 10mH	18,40
3H, 10mH	58,70
3H, 0.2H	73,20
3H, 0.2H, 10mH	74,80

Combination	Ω
40mH, 10mH, 2H	61,30
20mH, 30mH, 0.2H, 0.3H, 2H, 3H	150,40
0.1H, 20mH, 2H, 0.4H	91,00
10mH, 20mH, 0.1H, 0.2H, 1H, 2H	80,40
30mH, 40mH, 0.3H, 0.4H, 3H, 4H	143,40

Comparing the resulting resistances with individual resistances and pair resistances, a pattern can be deduced: non-linearity appears only for pairs 1+2 and 3+4, all other connections simply sum the resistances. That is, knowing all individual resistances and knowing the measured resistances of pairs 1+2 and 3+4 for each column - we can calculate the resistance of any combination of resistors. If switches 1 and 2 or 3 and 4 are present together in the combination - it is necessary not to sum up their individual resistances, but to take the measured pair resistance.

Finally, we can build the scheme for the entire device:

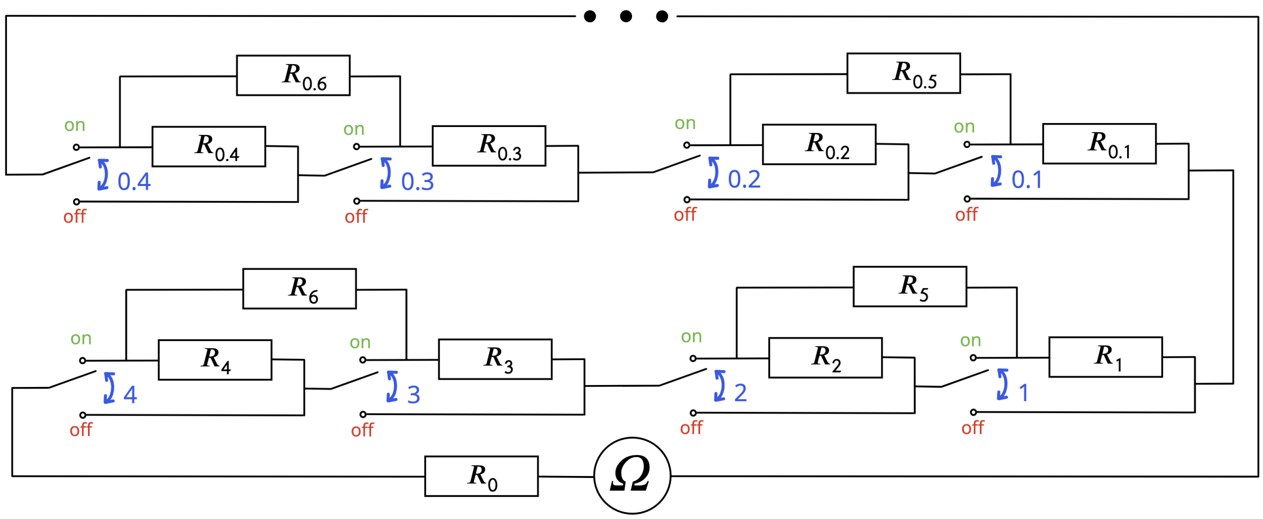

Figure 5: Scheme of the entire device. Indices 0.1,0.2,0.3... correspond to the column with tenths H, "..." corresponds to switches in the other columns, connected in series. Ⓘ represents an ohmmeter.

5 Next Steps

As the next step in our investigation, we will test a second inductance box to assess the applicability of the physical model developed from the first box. This will help determine whether the non-linear resistance behavior observed is consistent across different boxes.

6 Conclusion

Resistances were measured for individual switch activation, two-switch activation, and random activation of multiple switches. The statistical model was created to predict the resistance of every possible combination, knowing the individual resistances and the "nonlinear pairs" (1+2 and 3+4) resistances. The physical model was developed, and the proposed scheme of switches and resistances explains the nonlinear behavior of the device's resistance. The statistical and physical models were linked by deriving each coefficient of the statistical model using the resistances in the physical model.

A possible electrical scheme for the device was created. There are countless possible connections of different numbers of resistors that could yield the same measurement results, but the simplest connection was chosen from the available options, guided by Occam's razor principle.